

NAG Library Function Document

nag_reml_hier_mixed_regsn (g02jdc)

1 Purpose

nag_reml_hier_mixed_regsn (g02jdc) fits a multi-level linear mixed effects regression model using restricted maximum likelihood (REML). Prior to calling nag_reml_hier_mixed_regsn (g02jdc) the initialization function nag_hier_mixed_init (g02jcc) must be called.

2 Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_reml_hier_mixed_regsn (Integer lvpr, const Integer vpr[],
    Integer nvpr, double gamma[], Integer *effn, Integer *rnkx,
    Integer *ncov, double *lnlike, Integer lb, Integer id[], Integer pdid,
    double b[], double se[], double czz[], Integer pdczz, double cxx[],
    Integer pdcxx, double cxz[], Integer pdcxz, const double rcomm[],
    const Integer icomm[], const Integer iopt[], Integer liopt,
    const double ropt[], Integer lropt, NagError *fail)
```

3 Description

nag_reml_hier_mixed_regsn (g02jdc) fits a model of the form:

$$y = X\beta + Z\nu + \epsilon$$

where y is a vector of n observations on the dependent variable,

X is a known n by p design matrix for the *fixed* independent variables,

β is a vector of length p of unknown *fixed effects*,

Z is a known n by q design matrix for the *random* independent variables,

ν is a vector of length q of unknown *random effects*,

and ϵ is a vector of length n of unknown random errors.

Both ν and ϵ are assumed to have a Gaussian distribution with expectation zero and variance/covariance matrix defined by

$$\text{Var} \begin{bmatrix} \nu \\ \epsilon \end{bmatrix} = \begin{bmatrix} G & 0 \\ 0 & R \end{bmatrix}$$

where $R = \sigma_R^2 I$, I is the $n \times n$ identity matrix and G is a diagonal matrix. It is assumed that the random variables, Z , can be subdivided into $g \leq q$ groups with each group being identically distributed with expectation zero and variance σ_i^2 . The diagonal elements of matrix G therefore take one of the values $\{\sigma_i^2 : i = 1, 2, \dots, g\}$, depending on which group the associated random variable belongs to.

The model therefore contains three sets of unknowns: the fixed effects β , the random effects ν and a vector of $g + 1$ variance components γ , where $\gamma = \{\sigma_1^2, \sigma_2^2, \dots, \sigma_{g-1}^2, \sigma_g^2, \sigma_R^2\}$. Rather than working directly with γ , nag_reml_hier_mixed_regsn (g02jdc) uses an iterative process to estimate $\gamma^* = \{\sigma_1^2/\sigma_R^2, \sigma_2^2/\sigma_R^2, \dots, \sigma_{g-1}^2/\sigma_R^2, \sigma_g^2/\sigma_R^2, 1\}$. Due to the iterative nature of the estimation a set of initial values, γ_0 , for γ^* is required. nag_reml_hier_mixed_regsn (g02jdc) allows these initial values either to be supplied by you or calculated from the data using the minimum variance quadratic unbiased estimators (MIVQUE0) suggested by Rao (1972).

nag_reml_hier_mixed_regsn (g02jdc) fits the model by maximizing the restricted log-likelihood function:

$$-2l_R = \log(|V|) + (n-p)\log(r^T V^{-1} r) + \log|X^T V^{-1} X| + (n-p)(1 + \log(2\pi/(n-p)))$$

where

$$V = ZGZ^T + R, \quad r = y - Xb \quad \text{and} \quad b = (X^T V^{-1} X)^{-1} X^T V^{-1} y.$$

Once the final estimates for γ^* have been obtained, the value of σ_R^2 is given by

$$\sigma_R^2 = (r^T V^{-1} r)/(n-p).$$

Case weights, W_c , can be incorporated into the model by replacing $X^T X$ and $Z^T Z$ with $X^T W_c X$ and $Z^T W_c Z$ respectively, for a diagonal weight matrix W_c .

The log-likelihood, l_R , is calculated using the sweep algorithm detailed in Wolfinger *et al.* (1994).

4 References

- Goodnight J H (1979) A tutorial on the SWEEP operator *The American Statistician* **33(3)** 149–158
- Harville D A (1977) Maximum likelihood approaches to variance component estimation and to related problems *JASA* **72** 320–340
- Rao C R (1972) Estimation of variance and covariance components in a linear model *J. Am. Stat. Assoc.* **67** 112–115
- Stroup W W (1989) Predictable functions and prediction space in the mixed model procedure *Applications of Mixed Models in Agriculture and Related Disciplines Southern Cooperative Series Bulletin No. 343* 39–48
- Wolfinger R, Tobias R and Sall J (1994) Computing Gaussian likelihoods and their derivatives for general linear mixed models *SIAM Sci. Statist. Comput.* **15** 1294–1310

5 Arguments

Note: prior to calling nag_reml_hier_mixed_regsn (g02jdc) the initialization function nag_hier_mixed_init (g02jcc) must be called, therefore this documentation should be read in conjunction with the document for nag_hier_mixed_init (g02jcc).

In particular some argument names and conventions described in that document are also relevant here, but their definition has not been repeated. Specifically, **RNDM**, **wt**, **n**, **nff**, **nrf**, **nlsv**, **levels**, **fixed**, **DAT**, **licomm** and **lrcomm** should be interpreted identically in both functions.

1: **lvpr** – Integer *Input*

On entry: the sum of the number of random parameters and the random intercept flags specified in the call to nag_hier_mixed_init (g02jcc).

Constraint: $lvpr = \sum_i \mathbf{RNDM}(1, i) + \mathbf{RNDM}(2, i)$.

2: **vpr[lvpr]** – const Integer *Input*

On entry: a vector of flags indicating the mapping between the random variables specified in **rndm** and the variance components, σ_i^2 . See Section 9 for more details.

Constraint: $1 \leq vpr[i-1] \leq nvpr$, for $i = 1, 2, \dots, lvpr$.

3: **nvpr** – Integer *Input*

On entry: g , the number of variance components being estimated (excluding the overall variance, σ_R^2).

Constraint: $1 \leq nvpr \leq lvpr$.

- 4: **gamma**[**nvpr** + 1] – double *Input/Output*
On entry: holds the initial values of the variance components, γ_0 , with **gamma**[$i - 1$] the initial value for σ_i^2/σ_R^2 , for $i = 1, 2, \dots, \mathbf{nvpr}$.
 If **gamma**[0] = -1.0, the remaining elements of **gamma** are ignored and the initial values for the variance components are estimated from the data using MIVQUE0.
On exit: **gamma**[$i - 1$], for $i = 1, 2, \dots, \mathbf{nvpr}$, holds the final estimate of σ_i^2 and **gamma**[**nvpr**] holds the final estimate for σ_R^2 .
Constraint: **gamma**[0] = -1.0 or **gamma**[$i - 1$] ≥ 0.0 , for $i = 1, 2, \dots, g$.
- 5: **effn** – Integer * *Output*
On exit: effective number of observations. If there are no weights (i.e., **wt** is **NULL**), or all weights are nonzero, then **effn** = **n**.
- 6: **rnkx** – Integer * *Output*
On exit: the rank of the design matrix, X , for the fixed effects.
- 7: **ncov** – Integer * *Output*
On exit: number of variance components not estimated to be zero. If none of the variance components are estimated to be zero, then **ncov** = **nvpr**.
- 8: **lnlike** – double * *Output*
On exit: $-2l_R(\hat{\gamma})$ where l_R is the log of the restricted maximum likelihood calculated at $\hat{\gamma}$, the estimated variance components returned in **gamma**.
- 9: **lb** – Integer *Input*
On entry: the dimension of the arrays **b** and **se**.
Constraint: **lb** $\geq \mathbf{nff} + \mathbf{nrf} \times \mathbf{nlsv}$.
- 10: **id**[**pdid** \times **lb**] – Integer *Output*
Note: where **ID**(i, j) appears in this document, it refers to the array element **id**[($j - 1$) \times **pdid** + $i - 1$].
On exit: an array describing the parameter estimates returned in **b**. The first **nlsv** \times **nrf** columns of **ID** describe the parameter estimates for the random effects and the last **nff** columns the parameter estimates for the fixed effects.
 The example program for this function includes a demonstration of decoding the parameter estimates given in **b** using information from **id**.
 For fixed effects:
 for $l = \mathbf{nrf} \times \mathbf{nlsv} + 1, \dots, \mathbf{nrf} \times \mathbf{nlsv} + \mathbf{nff}$
 if **b**[$l - 1$] contains the parameter estimate for the intercept then

$$\mathbf{ID}(1, l) = \mathbf{ID}(2, l) = \mathbf{ID}(3, l) = 0;$$
 if **b**[$l - 1$] contains the parameter estimate for the i th level of the j th fixed variable, that is the vector of values held in the k th column of **DAT** when **fixed**[$j + 1$] = k then

$$\begin{aligned} \mathbf{ID}(1, l) &= 0, \\ \mathbf{ID}(2, l) &= j, \\ \mathbf{ID}(3, l) &= i; \end{aligned}$$
 if the j th variable is continuous or binary, that is **levels**[**fixed**[$j + 1$] - 1] = 1, then

$$\mathbf{ID}(3, l) = 0;$$

any remaining rows of the l th column of **ID** are set to 0.

For random effects:

let

N_{R_b} denote the number of random variables in the b th random statement, that is $N_{R_b} = \mathbf{RNDM}(1, b)$;

R_{jb} denote the j th random variable from the b th random statement, that is the vector of values held in the k th column of **DAT** when $\mathbf{RNDM}(2 + j, b) = k$;

N_{S_b} denote the number of subject variables in the b th random statement, that is $N_{S_b} = \mathbf{RNDM}(3 + N_{R_b}, b)$;

S_{jb} denote the j th subject variable from the b th random statement, that is the vector of values held in the k th column of **DAT** when $\mathbf{RNDM}(3 + N_{R_b} + j, b) = k$;

$L(S_{jb})$ denote the number of levels for S_{jb} , that is $L(S_{jb}) = \mathbf{levels}[\mathbf{RNDM}(3 + N_{R_b} + j, b) - 1]$;

then

for $l = 1, 2, \dots, \mathbf{nrf} \times \mathbf{nlsv}$, if $\mathbf{b}[l - 1]$ contains the parameter estimate for the i th level of R_{jb} when $S_{kb} = s_k$, for $k = 1, 2, \dots, N_{S_b}$ and $1 \leq s_k \leq L(S_{jb})$, i.e., s_k is a valid value for the k th subject variable, then

$$\begin{aligned} \mathbf{ID}(1, l) &= b, \\ \mathbf{ID}(2, l) &= j, \\ \mathbf{ID}(3, l) &= i, \\ \mathbf{ID}(3 + k, l) &= s_k, \quad k = 1, 2, \dots, N_{S_b}; \end{aligned}$$

if the parameter being estimated is for the intercept then $\mathbf{ID}(2, l) = \mathbf{ID}(3, l) = 0$;

if the j th variable is continuous, or binary, that is $L(S_{jb}) = 1$, then $\mathbf{ID}(3, l) = 0$;

the remaining rows of the l th column of **ID** are set to 0.

In some situations, certain combinations of variables are never observed. In such circumstances all elements of the l th row of **ID** are set to -999 .

11: **pdid** – Integer *Input*

On entry: the stride separating matrix row elements in the array **id**.

Constraint: $\mathbf{pdid} \geq 3 + \max_j(\mathbf{RNDM}(3 + \mathbf{RNDM}(1, j), j))$, i.e., $3 +$ maximum number of subject variables (see `nag_hier_mixed_init` (g02jcc)).

12: **b[*lb*]** – double *Output*

On exit: the parameter estimates, with the first $\mathbf{nrf} \times \mathbf{nlsv}$ elements of **b** containing the parameter estimates for the random effects, ν , and the remaining **nff** elements containing the parameter estimates for the fixed effects, β . The order of these estimates are described by the **id** argument.

13: **se[*lb*]** – double *Output*

On exit: the standard errors of the parameter estimates given in **b**.

14: **czz[*dim*]** – double *Output*

Note: the dimension, *dim*, of the array **czz** must be at least $\mathbf{pdczz} \times \mathbf{nrf} \times \mathbf{nlsv}$.

Where $\mathbf{CZZ}(i, j)$ appears in this document, it refers to the array element $\mathbf{czz}[(j - 1) \times \mathbf{pdczz} + i - 1]$.

On exit: if $\mathbf{nlsv} = 1$, then **CZZ** holds the lower triangular portion of the matrix $(1/\sigma^2)(Z^T \hat{R}^{-1} Z + \hat{G}^{-1})$, where \hat{R} and \hat{G} are the estimates of R and G respectively. If

$\mathbf{nlsv} > 1$ then **CZZ** holds this matrix in compressed form, with the first **nrf** columns holding the part of the matrix corresponding to the first level of the overall subject variable, the next **nrf** columns the part corresponding to the second level of the overall subject variable etc.

15: **pdzcz** – Integer *Input*

On entry: the stride separating matrix row elements in the array **czz**.

Constraint: **pdzcz** \geq **nrf**.

16: **cx**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **cx** must be at least **pdccx** \times **nff**.

Where **CXX**(*i*, *j*) appears in this document, it refers to the array element **cx**[(*j* – 1) \times **pdccx** + *i* – 1].

On exit: **CXX** holds the lower triangular portion of the matrix $(1/\sigma^2)X^T\hat{V}^{-1}X$, where \hat{V} is the estimated value of *V*.

17: **pdccx** – Integer *Input*

On entry: the stride separating matrix row elements in the array **cx**.

Constraint: **pdccx** \geq **nff**.

18: **cxz**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **cxz** must be at least **pdccz** \times **nlsv** \times **nrf**.

Where **CXZ**(*i*, *j*) appears in this document, it refers to the array element **cxz**[(*j* – 1) \times **pdccz** + *i* – 1].

On exit: if **nlsv** = 1, then **CXZ** holds the matrix $(1/\sigma^2)(X^T\hat{V}^{-1}Z)\hat{G}$, where \hat{V} and \hat{G} are the estimates of *V* and *G* respectively. If **nlsv** > 1 then **CXZ** holds this matrix in compressed form, with the first **nrf** columns holding the part of the matrix corresponding to the first level of the overall subject variable, the next **nrf** columns the part corresponding to the second level of the overall subject variable etc.

19: **pdccz** – Integer *Input*

On entry: the stride separating matrix row elements in the array **czz**.

Constraint: **pdccz** \geq **nff**.

20: **rcomm**[*dim*] – const double *Communication Array*

Note: the dimension, *dim*, of the array **rcomm** must be at least **lrcomm**.

On entry: communication array initialized by a call to nag_hier_mixed_init (g02jcc).

21: **icomm**[*dim*] – const Integer *Communication Array*

Note: the dimension, *dim*, of the array **icomm** must be at least **licomm**.

On entry: communication array initialized by a call to nag_hier_mixed_init (g02jcc).

22: **iopt**[**liopt**] – const Integer *Input*

On entry: optional parameters passed to the optimization function.

By default nag_reml_hier_mixed_regsn (g02jdc) fits the specified model using a modified Newton optimization algorithm as implemented in the NAG Fortran Library routine E04LBF. In some cases, where the calculation of the derivatives is computationally expensive it may be more efficient to use a sequential QP algorithm. The sequential QP algorithm as implemented in the

NAG Fortran Library routine E04UCF can be chosen by setting $\mathbf{iopt}[4] = 1$. If $\mathbf{liopt} < 5$ or $\mathbf{iopt}[4] \neq 1$ then E04LBF will be used.

Different optional parameters are available depending on the optimization function used. In all cases, using a value of -1 will cause the default value to be used. In addition only the first \mathbf{liopt} values of \mathbf{iopt} are used, so for example, if only the first element of \mathbf{iopt} needs changing and default values for all other optional parameters are sufficient \mathbf{liopt} can be set to 1.

NAG Fortran Library routine E04LBF is being used.

<i>i</i>	Description	Equivalent E04LBF argument	Default Value
0	Number of iterations	MAXCAL	1000
1	Unit number for monitoring information. See nag_o_pen_file (x04acc) for details on how to assign a file to a unit number.	n/a	Output sent to stdout
2	Print optional parameters (1 = print)	n/a	-1 (no printing performed)
3	Frequency that monitoring information is printed	IPRINT	-1
4	Optimizer used	n/a	n/a

If requested, monitoring information is displayed in a similar format to that given by E04LBF.

NAG Fortran Library routine E04UCF is being used.

<i>i</i>	Description	Equivalent E04UCF argument	Default Value
0	Number of iterations	Major Iteration Limit	$\max(50, 3 \times \mathbf{nvpr})$
1	Unit number for monitoring information. See nag_o_pen_file (x04acc) for details on how to assign a file to a unit number.	n/a	Output sent to stdout
2	Print optional parameters (1 = print, otherwise no print)	List/ NoList	-1 (no printing performed)
3	Frequency that monitoring information is printed	Major Print Level	0
4	Optimizer used	n/a	n/a
5	Number of minor iterations	Minor Iteration Limit	$\max(50, 3 \times \mathbf{nvpr})$
6	Frequency that additional monitoring information is printed	Minor Print Level	0

If $\mathbf{liopt} \leq 0$ then default values are used for all optional parameters and \mathbf{iopt} may be set to NULL.

23: **liopt** – Integer *Input*

On entry: length of the options array **iopt**.

24: **ropt[liopt]** – const double *Input*

On entry: optional parameters passed to the optimization function.

Different optional parameters are available depending on the optimization function used. In all cases, using a value of -1.0 will cause the default value to be used. In addition only the first \mathbf{liopt} values of **ropt** are used, so for example, if only the first element of **ropt** needs changing and default values for all other optional parameters are sufficient **liopt** can be set to 1.

NAG Fortran Library routine E04LBF is being used.

<i>i</i>	Description	Equivalent E04LBF argument	Default Value
0	Sweep tolerance	n/a	$\max\left(\sqrt{\epsilon_{ps}}, \sqrt{\epsilon_{ps}} \times \max_i(zz_{ii})\right)$
1	Lower bound for γ^*	n/a	eps/100
2	Upper bound for γ^*	n/a	10^{20}

3	Accuracy of linear minimizations	ETA	0.9
4	Accuracy to which solution is required	XTOL	0.0
5	Initial distance from solution	STEMPX	100000.0

NAG Fortran Library routine E04UCF is being used.

<i>i</i>	Description	Equivalent E04UCF argument	Default Value
0	Sweep tolerance	n/a	$\max\left(\sqrt{\text{eps}}, \sqrt{\text{eps}} \times \max_i(\text{zz}_{ii})\right)$
1	Lower bound for γ^*	n/a	eps/100
2	Upper bound for γ^*	n/a	10^{20}
3	Line search tolerance	Line Search Tolerance	0.9
4	Optimality tolerance	Optimality Tolerance	$\text{eps}^{0.72}$

where eps is the *machine precision* returned by nag_machine_precision (X02AJC) and zz_{ii} denotes the *i* diagonal element of $Z^T Z$.

If **iropt** ≤ 0 then default values are used for all optional parameters and **ropt** and may be set to NULL.

25: **iropt** – Integer *Input*

On entry: length of the options array **ropt**.

26: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle \text{value} \rangle$ had an illegal value.

NE_INT

On entry, **lb** = $\langle \text{value} \rangle$.

Constraint: **lb** $\geq \langle \text{value} \rangle$.

On entry, **lvpr** = $\langle \text{value} \rangle$.

Constraint: **lvpr** $\geq \langle \text{value} \rangle$.

On entry, **nvpr** = $\langle \text{value} \rangle$.

Constraint: $1 \leq \text{nvpr} \leq \langle \text{value} \rangle$.

On entry, **pdcxx** = $\langle \text{value} \rangle$.

Constraint: **pdcxx** $\geq \langle \text{value} \rangle$.

On entry, **pdcxz** = $\langle \text{value} \rangle$.

Constraint: **pdcxz** $\geq \langle \text{value} \rangle$.

On entry, **pdczz** = $\langle \text{value} \rangle$.

Constraint: **pdczz** $\geq \langle \text{value} \rangle$.

On entry, **pdid** = $\langle value \rangle$.
 Constraint: **pdid** $\geq \langle value \rangle$.

NE_INT_ARRAY

On entry, at least one value of i , for $i = 1, 2, \dots, \mathbf{nvpr}$, does not appear in **vpr**.

On entry, **icomm** has not been initialized correctly.

On entry, **vpr**[$\langle value \rangle$] = $\langle value \rangle$ and **nvpr** = $\langle value \rangle$.
 Constraint: $1 \leq \mathbf{vpr}[i - 1] \leq \mathbf{nvpr}$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NEG_ELEMENT

At least one negative estimate for **gamma** was obtained. All negative estimates have been set to zero.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL_ARRAY

On entry, **gamma**[$\langle value \rangle$] = $\langle value \rangle$.
 Constraint: **gamma**[0] = -1.0 or **gamma**[$i - 1$] ≥ 0.0 .

NW_KT_CONDITIONS

Current point cannot be improved upon.

NW_NOT_CONVERGED

Optimal solution found, but requested accuracy not achieved.

NW_TOO_MANY_ITER

Too many major iterations.

7 Accuracy

Not applicable.

8 Parallelism and Performance

`nag_reml_hier_mixed_regsn` (g02jdc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_reml_hier_mixed_regsn` (g02jdc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The argument **vpr** gives the mapping between the random variables and the variance components. In most cases $\mathbf{vpr}[i - 1] = i$, for $i = 1, 2, \dots, \sum_i \mathbf{RNDM}(1, i) + \mathbf{RNDM}(2, i)$. However, in some cases it might be necessary to associate more than one random variable with a single variance component, for example, when the columns of **DAT** hold dummy variables.

Consider a dataset with three variables:

$$\mathbf{DAT} = \begin{pmatrix} 1 & 1 & 3.6 \\ 2 & 1 & 4.5 \\ 3 & 1 & 1.1 \\ 1 & 2 & 8.3 \\ 2 & 2 & 7.2 \\ 3 & 2 & 6.1 \end{pmatrix}$$

where the first column corresponds to a categorical variable with three levels, the next to a categorical variable with two levels and the last column to a continuous variable. So in a call to `nag_hier_mixed_init (g02jcc)`

$$\mathbf{levels} = (3 \quad 2 \quad 1)$$

also assume a model with no fixed effects, no random intercept, no nesting and all three variables being included as random effects, then

$$\begin{aligned} \mathbf{fixed} &= (0 \quad 0); \\ \mathbf{RNDM} &= (3 \quad 0 \quad 1 \quad 2 \quad 3)^T. \end{aligned}$$

Each of the three columns in **DAT** therefore correspond to a single variable and hence there are three variance components, one for each random variable included in the model, so

$$\mathbf{vpr} = (1 \quad 2 \quad 3).$$

This is the recommended way of supplying the data to `nag_reml_hier_mixed_regsn (g02jdc)`, however it is possible to reformat the above dataset by replacing each of the categorical variables with a series of dummy variables, one for each level. The dataset then becomes

$$\mathbf{DAT} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 3.6 \\ 0 & 1 & 0 & 1 & 0 & 4.5 \\ 0 & 0 & 1 & 1 & 0 & 1.1 \\ 1 & 0 & 0 & 0 & 1 & 8.3 \\ 0 & 1 & 0 & 0 & 1 & 7.2 \\ 0 & 0 & 1 & 0 & 1 & 6.1 \end{pmatrix}$$

where each column only has one level

$$\mathbf{levels} = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1).$$

Again a model with no fixed effects, no random intercept, no nesting and all variables being included as random effects is required, so

$$\begin{aligned} \mathbf{fixed} &= (0 \quad 0); \\ \mathbf{RNDM} &= (6 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6)^T. \end{aligned}$$

With the data entered in this manner, the first three columns of **DAT** correspond to a single variable (the first column of the original dataset) as do the next two columns (the second column of the original dataset). Therefore **vpr** must reflect this

$$\mathbf{vpr} = (1 \quad 1 \quad 1 \quad 2 \quad 2 \quad 3).$$

In most situations it is more efficient to supply the data to `nag_hier_mixed_init (g02jcc)` in terms of categorical variables rather than transform them into dummy variables.

10 Example

This example fits a random effects model with three levels of nesting to a simulated dataset with 90 observations and 12 variables.

10.1 Program Text

```

/* nag_reml_hier_mixed_regsn (g02jdc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg02.h>

void print_results(Nag_OrderType order, Integer n, Integer nff, Integer nlsv,
                  Integer nrf, Integer fixed[], Integer nrndm,
                  Integer rndm[], Integer lrndm, Integer nvpr,
                  Integer vpr[], double gamma[], Integer effn,
                  Integer rnkx, Integer ncov, double lnlike,
                  Integer id[], Integer pdid, double b[], double se[]);

#define RNDM(I, J) rndm[(order == Nag_ColMajor) \
                       ?((J-1)*lrndm+I-1):((I-1)*nrndm+J-1)]
#define DAT(I, J)  dat[(order == Nag_ColMajor) \
                       ?((J-1)*pddat+I-1):((I-1)*pddat+J-1)]
#define ID(I, J)   id[((J-1)*pdid+I-1)]

int main(void)
{
    /* IO file pointers */

    /* Integer scalar and array declarations */
    Integer exit_status = 0;
    Integer pdid, licomm, lrcomm, tdczz, lb, pdcxx, pdcxz, pdczz, pddat,
           effn, i, j, lvpr, n, ncol, ncov, lfixed, nff, nl, nlsv, nrndm,
           nrf, nv, nvpr, rnkx, lwt, size_dat, lrndm;
    Integer *fixed = 0, *icomm = 0, *id = 0, *levels = 0, *rndm = 0;
    Integer *vpr = 0;
    Integer ticomm[2];

    /* NAG structures */
    NagError fail;
    Nag_OrderType order = Nag_RowMajor;

    /* Double scalar and array declarations */
    double lnlike;
    double *b = 0, *cxx = 0, *cxz = 0, *czz = 0, *dat = 0, *gamma = 0;
    double *rcomm = 0, *se = 0, *wt = 0, *y = 0;
    double trcomm[1];

    /* Character scalars */
    char weight;

    /* Use the default options */
    Integer *iopt = 0;
    Integer liopt = 0;
    double *ropt = 0;
    Integer lropt = 0;

    /* Initialize the error structure */
    INIT_FAIL(fail);

```

```

printf("nag_reml_hier_mixed_regsn (g02jdc) Example Program Results\n\n");

/* Skip headings in data file */
#ifdef _WIN32
scanf_s("%*[\n] ");
#else
scanf("%*[\n] ");
#endif

/* Read in the initial arguments */
#ifdef _WIN32
scanf_s("%c%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ",
        &weight, 1, &n, &ncol, &nrndm, &nvpr);
#else
scanf("%c%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ",
        &weight, &n, &ncol, &nrndm, &nvpr);
#endif

/* Maximum size for fixed and rndm */
lfixed = ncol + 2;
lrndm = 2 * ncol + 3;

if (order == Nag_ColMajor) {
    pddat = n;
    size_dat = pddat * ncol;
}
else {
    pddat = ncol;
    size_dat = pddat * n;
}

/* Allocate some memory */
if (!(y = NAG_ALLOC(n, double)) ||
    !(vpr = NAG_ALLOC(nvpr, Integer)) ||
    !(levels = NAG_ALLOC(ncol, Integer)) ||
    !(gamma = NAG_ALLOC(nvpr + 1, double)) ||
    !(fixed = NAG_ALLOC(lfixed, Integer)) ||
    !(rndm = NAG_ALLOC(lrndm * nrndm, Integer)) ||
    !(dat = NAG_ALLOC(size_dat, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Check whether we are supplying weights and
   allocate memory if required */
if (weight == 'W') {
    lwt = n;
    if (!(wt = NAG_ALLOC(lwt, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
}
else {
    lwt = 0;
}

/* Read in the number of levels associated with each of the
   independent variables */
for (i = 0; i < ncol; i++)
#ifdef _WIN32
scanf_s("%" NAG_IFMT "", &levels[i]);
#else
scanf("%" NAG_IFMT "", &levels[i]);
#endif
#ifdef _WIN32
scanf_s("%*[\n] ");

```

```

#else
    scanf("%*[\n] ");
#endif

    /* Read in the fixed part of the model */
    /* Skip the heading */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    /* Number of variables */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &fixed[0]);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &fixed[0]);
#endif
    nv = fixed[0];
    if (nv + 2 > lfixed) {
        printf(" ** Problem size too large, increase array sizes\n");
        printf("LFIXED,NV+2 = %" NAG_IFMT " , %" NAG_IFMT "\n", lfixed, nv + 2);
        exit_status = -1;
        goto END;
    }
    /* Intercept */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &fixed[1]);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &fixed[1]);
#endif
    /* Variable IDs */
    if (nv > 0) {
        for (i = 2; i < nv + 2; i++)
#ifdef _WIN32
            scanf_s("%" NAG_IFMT "", &fixed[i]);
#else
            scanf("%" NAG_IFMT "", &fixed[i]);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}

    /* Read in the random part of the model */
    lvpr = 0;
    pdid = 0;
    for (j = 1; j <= nrndm; j++) {
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        scanf("%*[\n] ");
#endif
    }
    /* Number of variables */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &RNDM(1, j));
#else
    scanf("%" NAG_IFMT "%*[\n] ", &RNDM(1, j));
#endif
    nv = RNDM(1, j);
    if ((nv + 3) > lrndm) {
        printf(" ** Problem size too large, increase array sizes\n");
        printf("LRNDM,NV+2 = %" NAG_IFMT " , %" NAG_IFMT "\n", lrndm, nv + 2);
        exit_status = -1;
        goto END;
    }
    /* Intercept */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &RNDM(2, j));
#else

```

```

        scanf("%" NAG_IFMT "%*[\n] ", &RNDM(2, j));
#endif
    /* Variable IDs */
    if (nv > 0) {
        for (i = 3; i <= nv + 2; i++)
#ifdef _WIN32
            scanf_s("%" NAG_IFMT "", &RNDM(i, j));
#else
            scanf("%" NAG_IFMT "", &RNDM(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    }
    /* Number of subject variables */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &RNDM(nv + 3, j));
#else
    scanf("%" NAG_IFMT "%*[\n] ", &RNDM(nv + 3, j));
#endif
    nl = RNDM(nv + 3, j);
    if (nv + nl + 2 > lrndm) {
        printf(" ** Problem size too large, increase array sizes\n");
        printf("LRNDM,NV+NL+2 = %" NAG_IFMT " , %" NAG_IFMT "\n",
            lrndm, nv + nl + 2);
        exit_status = -1;
        goto END;
    }
    /* Subject variable IDs */
    if (nl > 0) {
        for (i = nv + 4; i <= nv + nl + 3; i++)
#ifdef _WIN32
            scanf_s("%" NAG_IFMT "", &RNDM(i, j));
#else
            scanf("%" NAG_IFMT "", &RNDM(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    }
    pdid = MAX(pdid, nl);
    lvpr += RNDM(2, j) + nv;
}
pdid += 3;

    /* Read in the dependent and independent data */
    for (i = 1; i <= n; i++) {
#ifdef _WIN32
        scanf_s("%lf", &y[i - 1]);
#else
        scanf("%lf", &y[i - 1]);
#endif
        for (j = 1; j <= ncol; j++)
#ifdef _WIN32
            scanf_s("%lf", &DAT(i, j));
#else
            scanf("%lf", &DAT(i, j));
#endif
        if (lwt > 0)
#ifdef _WIN32
            scanf_s("%lf", &wt[i - 1]);
#else
            scanf("%lf", &wt[i - 1]);
#endif
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        }
    }

```

```

    scanf("%*[\n] ");
#endif
}

/* Read in VPR */
for (i = 0; i < lvpr; i++)
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &vpr[i]);
#else
    scanf("%" NAG_IFMT "", &vpr[i]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Read in GAMMA */
for (i = 0; i < nvpr; i++)
#ifdef _WIN32
    scanf_s("%lf", &gamma[i]);
#else
    scanf("%lf", &gamma[i]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Get the size of the communication arrays */
licomm = 2;
lrcomm = 1;
nag_hier_mixed_init(order, n, ncol, dat, pddat, levels, y, wt, fixed,
                    lfixed, nrndm, rndm, lrndm, &nff, &nlsv, &nrf, trcomm,
                    lrcomm, ticomm, licomm, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_hier_mixed_init (g02jcc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
licomm = ticomm[0];
lrcomm = ticomm[1];

/* Allocate the communication arrays */
if (!(icomm = NAG_ALLOC(licomm, Integer)) ||
    !(rcomm = NAG_ALLOC(lrcomm, double)))
{
    printf("Allocation failure 4\n");
    exit_status = -1;
    goto END;
}

/* Pre-process the data */
nag_hier_mixed_init(order, n, ncol, dat, pddat, levels, y, wt, fixed,
                    lfixed, nrndm, rndm, lrndm, &nff, &nlsv, &nrf, rcomm,
                    lrcomm, icomm, licomm, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_hier_mixed_init (g02jcc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Allocate the output arrays */
lb = nff + nrf * nlsv;
tdczz = nrf * nlsv;
pdcxx = nff;
pdcxz = nff;
pdczz = nrf;
if (!(b = NAG_ALLOC(lb, double)) ||
    !(cxx = NAG_ALLOC(pdcxx * nff, double)) ||

```

```

        !(cxz = NAG_ALLOC(pdcxz * tdczz, double)) ||
        !(czz = NAG_ALLOC(pdczz * tdczz, double)) ||
        !(se = NAG_ALLOC(lb, double)) || !(id = NAG_ALLOC(pdid * lb, Integer)))
    {
        printf("Allocation failure 5\n");
        exit_status = -1;
        goto END;
    }

    /* Perform the analysis */
    nag_reml_hier_mixed_regsn(lvpr, vpr, nvpr, gamma, &effn, &rnkx, &ncov,
                             &lnlike, lb, id, pdid, b, se, czz, pdczz, cxx,
                             pdcxx, cxz, pdcxz, rcomm, icomm, iopt, liopt,
                             ropt, lropt, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_reml_hier_mixed_regsn (g02jdc).\n%s\n",
              fail.message);
        exit_status = 1;
        if (fail.code != NW_NOT_CONVERGED && fail.code != NW_TOO_MANY_ITER &&
            fail.code != NW_KT_CONDITIONS && fail.code != NE_NEG_ELEMENT)
            goto END;
    }

    /* Display the output */
    print_results(order, n, nff, nlsv, nrf, fixed, nrndm, rndm, lrndm, nvpr,
                 vpr, gamma, effn, rnkx, ncov, lnlike, id, pdid, b, se);

END:

    NAG_FREE(wt);
    NAG_FREE(y);
    NAG_FREE(vpr);
    NAG_FREE(levels);
    NAG_FREE(gamma);
    NAG_FREE(fixed);
    NAG_FREE(rndm);
    NAG_FREE(dat);
    NAG_FREE(icomm);
    NAG_FREE(rcomm);
    NAG_FREE(b);
    NAG_FREE(cxx);
    NAG_FREE(cxz);
    NAG_FREE(czz);
    NAG_FREE(se);
    NAG_FREE(id);

    return exit_status;
}

void print_results(Nag_OrderType order, Integer n, Integer nff, Integer nlsv,
                  Integer nrf, Integer fixed[], Integer nrndm,
                  Integer rndm[], Integer lrndm, Integer nvpr,
                  Integer vpr[], double gamma[], Integer effn,
                  Integer rnkx, Integer ncov, double lnlike,
                  Integer id[], Integer pdid, double b[], double se[])
{
    Integer aid, i, k, l, ns, nv, p, pb, tb, tdid, vid, same;

    /* Display the output */
    printf(" Number of observations (N)                = %" NAG_IFMT "\n",
           n);
    printf(" Number of random factors (NRF)           = %" NAG_IFMT "\n",
           nrf);
    printf(" Number of fixed factors (NFF)                   = %" NAG_IFMT "\n",
           nff);
    printf(" Number of subject levels (NLSV)                  = %" NAG_IFMT "\n",
           nlsv);
    printf(" Rank of X (RNKX)                                  = %" NAG_IFMT "\n",
           rnkx);
    printf(" Effective N (EFFN)                                = %" NAG_IFMT "\n",
           effn);

```

```

printf(" Number of nonzero variance components (NCOV) = %" NAG_IFMT "\n",
      ncov);

printf(" Parameter Estimates\n");
tdid = nff + nrf * nlsv;

if (nrf > 0) {
  printf("\n");
  printf(" Random Effects\n");
}

pb = -999;
for (k = 1; k <= nrf * nlsv; k++) {
  tb = ID(1, k);
  if (tb != -999) {
    vid = ID(2, k);
    nv = RNDM(1, tb);
    ns = RNDM(3 + nv, tb);

    if (pb != tb) {
      same = 0;
    }
    else {
      same = 1;
      for (l = 1; l <= ns; l++) {
        if (ID(3 + l, k) != ID(3 + l, k - 1)) {
          same = 0;
          break;
        }
      }
    }

    if (!same) {
      if (k != 1)
        printf("\n");
      printf(" Subject: ");
      for (l = 1; l <= ns; l++)
        printf(" Variable %2" NAG_IFMT " (Level %1" NAG_IFMT " ) ",
              RNDM(3 + nv + l, tb), ID(3 + l, k));
      printf("\n");
    }
    pb = tb;

    if (vid == 0) {
      /* Intercept */
      printf("      Intercept                %10.4f %10.4f\n", b[k], se[k]);
    }
    else {
      /* VID'th variable specified in RNDM */
      aid = RNDM(2 + vid, tb);
      if (ID(3, k) == 0) {
        printf("      Variable %2" NAG_IFMT "", aid);
        printf("                %10.4f %10.4f\n", b[k - 1], se[k - 1]);
      }
      else {
        printf("      Variable %2" NAG_IFMT "", aid);
        printf(" (Level %1" NAG_IFMT " )                %10.4f %10.4f\n",
              ID(3, k), b[k - 1], se[k - 1]);
      }
    }
  }
}

if (nff > 0) {
  printf("\n");
  printf(" Fixed Effects\n");
}

for (k = nrf * nlsv + 1; k <= tdid; k++) {
  vid = ID(2, k);
  if (vid != -999) {
    if (vid == 0) {

```



```

    /* Intercept */
    printf("    Intercept                %10.4f %10.4f\n",
           b[k - 1], se[k - 1]);
}
else {
    /* VID'th variable specified in FIXED */
    aid = fixed[2 + vid - 1];
    if (ID(3, k) == 0) {
        printf("    Variable %2" NAG_IFMT " ", aid);
        printf("                %10.4f %10.4f\n", b[k - 1], se[k - 1]);
    }
    else {
        printf("    Variable %2" NAG_IFMT " ", aid);
        printf(" (Level %1" NAG_IFMT "                %10.4f %10.4f\n",
               ID(3, k), b[k - 1], se[k - 1]);
    }
}
}
}

printf("\n");
printf(" Variance Components\n");
printf(" Estimate      Parameter      Subject\n");
for (k = 1; k <= nvpr; k++) {
    printf("%10.5f      ", gamma[k - 1]);
    p = 0;
    for (tb = 1; tb <= nrndm; tb++) {
        nv = RNDM(1, tb);
        ns = RNDM(3 + nv, tb);
        for (i = 1; i <= nv + RNDM(2, tb); i++) {
            p++;
            if (vpr[p - 1] == k) {
                printf("Variable %2" NAG_IFMT "      Variables ", RNDM(2 + i, tb));
                for (l = 1; l <= ns; l++)
                    printf("%2" NAG_IFMT " ", RNDM(3 + nv + l, tb));
            }
        }
    }
    printf("\n");
}
printf("\n");
printf("SIGMA**2          = %15.5f\n", gamma[nvpr]);
printf("-2LOG LIKELIHOOD = %15.5f\n", lnlike);
}

```

10.2 Program Data

```

nag_reml_hier_mixed_regsn (g02jdc) Example Program Data
U 90 12 3 7      :: WEIGHT (U = no weights),N,NCOL,NRAND,NVPR
2 3 2 3 2 3 1 4 5 2 3 3  :: LEVELS(1:NCOL)
## FIXED
2                :: number of variables
1                :: intercept
1 2             :: variable IDs
## RANDOM 1
2                :: number of variables
0                :: intercept
3 4             :: variable IDs
3                :: number of subject variables
10 11 12        :: subject variable IDs
## RANDOM 2
2                :: number of variables
0                :: intercept
5 6             :: variable IDs
2                :: number of subject variables
11 12           :: subject variable IDs
## RANDOM 3
3                :: number of variables
0                :: intercept
7 8 9           :: variable IDs

```

```

1          :: number of subject variables
12         :: subject variable IDs
 3.1100  1.0 3.0 2.0 1.0 2.0 2.0 -0.3160 4.0 2.0 1.0 1.0 1.0
 2.8226  1.0 1.0 1.0 3.0 1.0 2.0 -1.3377 1.0 4.0 1.0 1.0 1.0
 7.4543  1.0 3.0 1.0 3.0 1.0 3.0 -0.7610 4.0 2.0 1.0 1.0 1.0
 4.4313  2.0 3.0 2.0 1.0 1.0 3.0 -2.2976 4.0 2.0 1.0 1.0 1.0
 6.1543  2.0 2.0 1.0 3.0 2.0 3.0 -0.4263 2.0 1.0 1.0 1.0 1.0
-0.1783  2.0 1.0 2.0 3.0 1.0 3.0  1.4067 3.0 3.0 2.0 1.0 1.0
 4.6748  2.0 3.0 2.0 1.0 2.0 1.0 -1.4669 1.0 2.0 2.0 1.0 1.0
 7.0667  1.0 1.0 1.0 3.0 2.0 3.0  0.4717 2.0 4.0 2.0 1.0 1.0
 1.4262  1.0 3.0 2.0 3.0 2.0 1.0  0.4436 1.0 3.0 2.0 1.0 1.0
 7.7290  1.0 1.0 1.0 2.0 2.0 3.0 -0.5950 3.0 4.0 2.0 1.0 1.0
-2.1806  1.0 3.0 1.0 3.0 1.0 1.0 -1.7981 4.0 2.0 1.0 2.0 1.0
 6.8419  2.0 3.0 1.0 2.0 1.0 1.0  0.2397 1.0 4.0 1.0 2.0 1.0
 1.2590  1.0 2.0 2.0 1.0 2.0 3.0  0.4742 1.0 1.0 1.0 2.0 1.0
 8.8405  2.0 2.0 2.0 2.0 2.0 3.0  0.6888 3.0 1.0 1.0 2.0 1.0
 6.1657  2.0 1.0 2.0 3.0 1.0 3.0 -1.0616 3.0 5.0 1.0 2.0 1.0
-4.5605  1.0 2.0 2.0 2.0 2.0 1.0 -0.5356 1.0 3.0 2.0 2.0 1.0
-1.2367  1.0 3.0 2.0 2.0 1.0 1.0 -1.2963 2.0 5.0 2.0 2.0 1.0
-12.2932 1.0 2.0 2.0 1.0 2.0 2.0 -1.5389 3.0 2.0 2.0 2.0 1.0
-2.3374  2.0 3.0 1.0 1.0 2.0 2.0 -0.6408 2.0 1.0 2.0 2.0 1.0
 0.0716  1.0 2.0 2.0 2.0 1.0 1.0  0.6574 1.0 1.0 2.0 2.0 1.0
 0.1895  2.0 1.0 1.0 1.0 1.0 3.0  0.9259 1.0 2.0 1.0 3.0 1.0
 1.5608  2.0 2.0 2.0 1.0 2.0 2.0  1.5080 3.0 1.0 1.0 3.0 1.0
-0.8529  2.0 3.0 1.0 1.0 1.0 3.0  2.5821 2.0 3.0 1.0 3.0 1.0
-4.1169  1.0 2.0 2.0 1.0 2.0 3.0  0.4102 1.0 4.0 1.0 3.0 1.0
 3.9977  2.0 1.0 2.0 3.0 2.0 2.0  0.7839 2.0 5.0 1.0 3.0 1.0
-8.1277  1.0 2.0 2.0 3.0 2.0 1.0 -1.8812 4.0 2.0 2.0 3.0 1.0
-4.9656  1.0 2.0 1.0 3.0 2.0 3.0  0.7770 4.0 1.0 2.0 3.0 1.0
-0.6428  2.0 2.0 1.0 2.0 1.0 3.0  0.2590 3.0 1.0 2.0 3.0 1.0
-5.5152  2.0 3.0 2.0 2.0 2.0 3.0 -0.9250 3.0 3.0 2.0 3.0 1.0
-5.5657  2.0 2.0 1.0 3.0 2.0 3.0 -0.4831 1.0 5.0 2.0 3.0 1.0
14.8177  2.0 2.0 1.0 3.0 1.0 3.0  0.5046 3.0 3.0 1.0 1.0 2.0
16.9783  2.0 1.0 1.0 2.0 2.0 1.0 -0.6903 2.0 1.0 1.0 1.0 2.0
13.8966  1.0 3.0 2.0 2.0 2.0 1.0  1.6166 2.0 5.0 1.0 1.0 2.0
14.8166  2.0 2.0 2.0 2.0 1.0 3.0  0.2778 2.0 3.0 1.0 1.0 2.0
19.3640  2.0 3.0 2.0 2.0 1.0 2.0  1.9586 4.0 2.0 1.0 1.0 2.0
 9.5299  1.0 3.0 1.0 1.0 1.0 3.0  1.0506 2.0 5.0 2.0 1.0 2.0
12.0102  2.0 1.0 1.0 3.0 2.0 3.0  0.4871 1.0 1.0 2.0 1.0 2.0
 6.1551  2.0 1.0 2.0 3.0 2.0 1.0  2.0891 4.0 4.0 2.0 1.0 2.0
-1.7048  1.0 2.0 1.0 1.0 2.0 2.0  1.4338 4.0 3.0 2.0 1.0 2.0
 2.7640  1.0 1.0 2.0 3.0 1.0 2.0 -1.1196 3.0 4.0 2.0 1.0 2.0
 2.8065  1.0 3.0 1.0 1.0 2.0 1.0  0.3367 3.0 2.0 1.0 2.0 2.0
 0.0974  2.0 2.0 1.0 3.0 1.0 1.0  0.1092 2.0 2.0 1.0 2.0 2.0
-7.8080  1.0 1.0 1.0 2.0 2.0 2.0  0.4007 4.0 1.0 1.0 2.0 2.0
-18.0450 2.0 3.0 1.0 1.0 1.0 2.0  0.1460 3.0 5.0 1.0 2.0 2.0
-2.8199  2.0 1.0 2.0 3.0 1.0 3.0 -0.3877 3.0 4.0 1.0 2.0 2.0
 8.9893  1.0 1.0 1.0 2.0 2.0 1.0  0.6957 4.0 3.0 2.0 2.0 2.0
 3.7978  2.0 1.0 1.0 1.0 2.0 1.0 -0.4664 3.0 3.0 2.0 2.0 2.0
-6.3493  1.0 1.0 1.0 1.0 2.0 3.0  0.2067 2.0 4.0 2.0 2.0 2.0
 8.1411  2.0 1.0 2.0 1.0 1.0 2.0  0.4112 1.0 4.0 2.0 2.0 2.0
-7.5483  2.0 2.0 1.0 1.0 1.0 2.0 -1.3734 3.0 3.0 2.0 2.0 2.0
-0.4600  2.0 1.0 2.0 3.0 1.0 3.0  0.7065 1.0 3.0 1.0 3.0 2.0
-3.2135  1.0 2.0 2.0 2.0 1.0 2.0  1.3628 4.0 2.0 1.0 3.0 2.0
-6.6562  2.0 1.0 2.0 2.0 2.0 3.0 -0.5052 4.0 5.0 1.0 3.0 2.0
 5.1267  2.0 1.0 1.0 1.0 2.0 1.0 -1.3457 2.0 5.0 1.0 3.0 2.0
 3.5592  1.0 1.0 2.0 1.0 2.0 3.0 -1.8022 3.0 4.0 1.0 3.0 2.0
-4.4420  2.0 3.0 1.0 2.0 1.0 1.0  0.0116 2.0 4.0 2.0 3.0 2.0
-8.5965  2.0 2.0 1.0 3.0 2.0 3.0 -0.9075 1.0 3.0 2.0 3.0 2.0
-6.3187  2.0 2.0 2.0 2.0 2.0 3.0 -1.4707 1.0 1.0 2.0 3.0 2.0
-7.8953  2.0 2.0 1.0 1.0 2.0 1.0 -1.2938 2.0 3.0 2.0 3.0 2.0
-10.1383 1.0 3.0 1.0 3.0 2.0 2.0 -1.1660 4.0 4.0 2.0 3.0 2.0
-7.8850  1.0 2.0 1.0 1.0 2.0 3.0  0.0397 4.0 4.0 1.0 1.0 3.0
23.2001  1.0 3.0 1.0 2.0 1.0 3.0 -0.5987 3.0 2.0 1.0 1.0 3.0
 5.5829  2.0 3.0 2.0 2.0 1.0 1.0  0.6683 3.0 3.0 1.0 1.0 3.0
-4.3698  2.0 2.0 1.0 1.0 2.0 2.0 -0.0106 1.0 3.0 1.0 1.0 3.0
 2.1274  1.0 2.0 1.0 3.0 2.0 2.0  0.5885 1.0 3.0 1.0 1.0 3.0
-2.7184  1.0 1.0 1.0 1.0 1.0 2.0  0.4555 1.0 5.0 2.0 1.0 3.0
-17.9128 2.0 2.0 2.0 1.0 1.0 2.0  0.6502 4.0 3.0 2.0 1.0 3.0
-1.2708  1.0 1.0 1.0 3.0 1.0 1.0 -0.1601 1.0 3.0 2.0 1.0 3.0
-24.2735 2.0 2.0 1.0 3.0 2.0 3.0  1.6910 1.0 1.0 2.0 1.0 3.0

```

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-14.7374 2.0 2.0 2.0 3.0 1.0 2.0 0.1053 4.0 4.0 2.0 1.0 3.0
 0.1713 2.0 1.0 2.0 3.0 2.0 2.0 -0.4037 3.0 4.0 1.0 2.0 3.0
 8.0006 1.0 3.0 2.0 3.0 1.0 3.0 -0.5853 3.0 2.0 1.0 2.0 3.0
 1.2100 2.0 3.0 2.0 1.0 1.0 1.0 -0.3037 1.0 3.0 1.0 2.0 3.0
 3.3307 1.0 3.0 1.0 1.0 2.0 2.0 -0.0774 1.0 4.0 1.0 2.0 3.0
-22.6713 2.0 3.0 1.0 2.0 2.0 1.0 0.4733 4.0 5.0 1.0 2.0 3.0
 7.5562 1.0 3.0 2.0 2.0 1.0 2.0 -0.0354 4.0 2.0 2.0 2.0 3.0
 -7.0694 1.0 3.0 2.0 2.0 1.0 1.0 -0.6640 2.0 1.0 2.0 2.0 3.0
 3.7159 2.0 3.0 1.0 3.0 1.0 1.0 0.0335 4.0 4.0 2.0 2.0 3.0
 -4.3135 1.0 2.0 2.0 2.0 1.0 3.0 0.1351 1.0 1.0 2.0 2.0 3.0
-14.5577 1.0 1.0 2.0 1.0 2.0 3.0 -0.5951 3.0 4.0 2.0 2.0 3.0
-12.5107 2.0 2.0 2.0 3.0 1.0 3.0 0.2735 3.0 2.0 1.0 3.0 3.0
 4.7708 2.0 2.0 1.0 1.0 1.0 3.0 0.3157 1.0 2.0 1.0 3.0 3.0
 13.2797 2.0 2.0 2.0 1.0 1.0 1.0 -1.0843 2.0 3.0 1.0 3.0 3.0
 -6.3243 1.0 2.0 2.0 1.0 2.0 2.0 -0.0836 4.0 2.0 1.0 3.0 3.0
 -7.0549 2.0 1.0 2.0 1.0 1.0 2.0 -0.2884 2.0 1.0 1.0 3.0 3.0
 -9.2713 2.0 3.0 2.0 3.0 2.0 3.0 -0.1006 1.0 2.0 2.0 3.0 3.0
-18.7788 1.0 3.0 1.0 2.0 2.0 3.0 0.5710 1.0 3.0 2.0 3.0 3.0
 -7.7230 1.0 1.0 2.0 1.0 1.0 2.0 0.2776 2.0 3.0 2.0 3.0 3.0
-22.7230 2.0 3.0 2.0 2.0 1.0 3.0 -0.7561 4.0 4.0 2.0 3.0 3.0
-11.6609 1.0 2.0 2.0 2.0 1.0 2.0 1.5549 1.0 4.0 2.0 3.0 3.0  :: Y, X
 1 2 3 4 5 6 7  :: VPR
-1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0  :: GAMMA(1:NVPR)

```

10.3 Program Results

nag_reml_hier_mixed_regsn (g02jdc) Example Program Results

```

Number of observations (N)           = 90
Number of random factors (NRF)      = 55
Number of fixed factors (NFF)       = 4
Number of subject levels (NLSV)     = 3
Rank of X (RNKX)                    = 4
Effective N (EFFN)                   = 90
Number of nonzero variance components (NCOV) = 7
Parameter Estimates

```

Random Effects

```

Subject: Variable 10 (Level 1) Variable 11 (Level 1) Variable 12 (Level 1)
Variable 3 (Level 1)          2.1561 3.7946
Variable 3 (Level 2)          1.8951 3.9284
Variable 4 (Level 1)          0.6496 3.1617

Subject: Variable 10 (Level 1) Variable 11 (Level 1) Variable 12 (Level 1)
Variable 4 (Level 3)          0.7390 3.1424

Subject: Variable 10 (Level 2) Variable 11 (Level 1) Variable 12 (Level 1)
Variable 3 (Level 1)          1.4216 3.3773
Variable 3 (Level 2)          -2.8921 3.3953
Variable 4 (Level 1)          3.6789 2.3162
Variable 4 (Level 2)          -1.9742 2.3887
Variable 4 (Level 3)          -2.2088 2.0697

Subject: Variable 10 (Level 1) Variable 11 (Level 2) Variable 12 (Level 1)
Variable 3 (Level 1)          -2.9659 3.9127
Variable 3 (Level 2)          2.7951 4.7183
Variable 4 (Level 1)          -4.7330 2.3094
Variable 4 (Level 2)          5.5161 2.2330
Variable 4 (Level 3)          -0.8417 2.3826

Subject: Variable 10 (Level 2) Variable 11 (Level 2) Variable 12 (Level 1)
Variable 3 (Level 1)          4.2202 3.6675
Variable 3 (Level 2)          -4.3883 3.4424
Variable 4 (Level 1)          -1.1391 3.2187
Variable 4 (Level 2)          1.0814 3.0654

Subject: Variable 10 (Level 1) Variable 11 (Level 3) Variable 12 (Level 1)
Variable 3 (Level 1)          0.3391 4.0647
Variable 3 (Level 2)          0.1502 3.4787
Variable 4 (Level 1)          -1.0026 2.4363

Subject: Variable 10 (Level 1) Variable 11 (Level 3) Variable 12 (Level 1)
Variable 4 (Level 3)          1.1703 2.6365

Subject: Variable 10 (Level 2) Variable 11 (Level 3) Variable 12 (Level 1)

```

Variable 3 (Level 1)	1.2658	3.4819	
Variable 3 (Level 2)	-1.5356	3.9097	
Subject: Variable 10 (Level 2)	Variable 11 (Level 3)	Variable 12 (Level 1)	
Variable 4 (Level 2)	0.7992	2.7902	
Variable 4 (Level 3)	-0.8916	2.8763	
Subject: Variable 11 (Level 1)	Variable 12 (Level 1)		
Variable 5 (Level 1)	-0.4885	2.8206	
Variable 5 (Level 2)	1.8829	2.7530	
Variable 6 (Level 1)	0.9249	3.7747	
Variable 6 (Level 2)	-2.3568	3.1624	
Variable 6 (Level 3)	4.3117	3.1474	
Subject: Variable 11 (Level 2)	Variable 12 (Level 1)		
Variable 5 (Level 1)	1.3898	2.9362	
Variable 5 (Level 2)	-1.5729	2.8909	
Variable 6 (Level 1)	0.2111	3.9967	
Variable 6 (Level 2)	-3.7083	4.2866	
Variable 6 (Level 3)	3.1190	4.7983	
Subject: Variable 11 (Level 3)	Variable 12 (Level 1)		
Variable 5 (Level 1)	1.7352	3.1370	
Variable 5 (Level 2)	-1.6165	3.1713	
Variable 6 (Level 1)	-1.1102	3.9374	
Variable 6 (Level 2)	4.4877	3.6980	
Variable 6 (Level 3)	-3.1325	3.1966	
Subject: Variable 12 (Level 1)			
Variable 7	0.6827	0.5060	
Variable 8 (Level 1)	1.5964	1.3206	
Variable 8 (Level 2)	-0.7533	1.5663	
Variable 8 (Level 3)	0.4035	1.6840	
Variable 8 (Level 4)	-0.8523	1.7518	
Variable 9 (Level 1)	0.5699	1.6236	
Variable 9 (Level 2)	0.0012	1.9111	
Variable 9 (Level 3)	-0.2850	1.9245	
Variable 9 (Level 4)	0.4468	2.0329	
Variable 9 (Level 5)	0.0030	2.1390	
Subject: Variable 10 (Level 1)	Variable 11 (Level 1)	Variable 12 (Level 2)	
Variable 3 (Level 1)	6.2551	3.3595	
Variable 3 (Level 2)	5.6085	3.4127	
Subject: Variable 10 (Level 1)	Variable 11 (Level 1)	Variable 12 (Level 2)	
Variable 4 (Level 2)	2.6922	2.7542	
Variable 4 (Level 3)	1.3742	2.8068	
Subject: Variable 10 (Level 2)	Variable 11 (Level 1)	Variable 12 (Level 2)	
Variable 3 (Level 1)	1.5647	3.8353	
Variable 3 (Level 2)	-2.7565	3.9041	
Variable 4 (Level 1)	-0.8621	2.8257	
Subject: Variable 10 (Level 2)	Variable 11 (Level 1)	Variable 12 (Level 2)	
Variable 4 (Level 3)	0.4536	2.8070	
Subject: Variable 10 (Level 1)	Variable 11 (Level 2)	Variable 12 (Level 2)	
Variable 3 (Level 1)	-10.1544	3.3433	
Variable 3 (Level 2)	3.2446	4.1221	
Variable 4 (Level 1)	-2.9419	2.3508	
Variable 4 (Level 2)	0.2510	3.0675	
Variable 4 (Level 3)	0.3224	2.9710	
Subject: Variable 10 (Level 2)	Variable 11 (Level 2)	Variable 12 (Level 2)	
Variable 3 (Level 1)	-1.3577	3.1925	
Variable 3 (Level 2)	8.1277	3.9975	
Variable 4 (Level 1)	-0.4290	2.4578	
Variable 4 (Level 2)	2.7495	2.5821	
Subject: Variable 10 (Level 1)	Variable 11 (Level 3)	Variable 12 (Level 2)	
Variable 3 (Level 1)	4.8432	4.0069	
Variable 3 (Level 2)	0.0370	3.6006	
Variable 4 (Level 1)	3.0713	2.2706	
Variable 4 (Level 2)	-1.8899	2.4756	
Variable 4 (Level 3)	0.4914	2.2914	
Subject: Variable 10 (Level 2)	Variable 11 (Level 3)	Variable 12 (Level 2)	
Variable 3 (Level 1)	-4.4766	3.3355	

Variable 3 (Level 2)	-3.7936	4.0759
Variable 4 (Level 1)	-0.5459	2.7097
Variable 4 (Level 2)	-1.5619	2.7412
Variable 4 (Level 3)	-0.7269	2.9735
Subject: Variable 11 (Level 1)	Variable 12 (Level 2)	
Variable 5 (Level 1)	4.8653	3.0706
Variable 5 (Level 2)	0.9011	3.0696
Variable 6 (Level 1)	6.9277	3.8411
Variable 6 (Level 2)	-1.3108	3.1667
Variable 6 (Level 3)	6.2916	3.5327
Subject: Variable 11 (Level 2)	Variable 12 (Level 2)	
Variable 5 (Level 1)	-0.4047	3.0956
Variable 5 (Level 2)	0.3291	3.0784
Variable 6 (Level 1)	6.9096	3.3073
Variable 6 (Level 2)	-1.0680	3.6213
Variable 6 (Level 3)	-5.9977	3.7299
Subject: Variable 11 (Level 3)	Variable 12 (Level 2)	
Variable 5 (Level 1)	-1.0925	3.0994
Variable 5 (Level 2)	-0.7392	2.9900
Variable 6 (Level 1)	2.7758	3.8748
Variable 6 (Level 2)	-6.3526	3.3014
Variable 6 (Level 3)	-0.2060	3.6481
Subject: Variable 12 (Level 2)		
Variable 7	0.1711	0.5785
Variable 8 (Level 1)	1.7186	1.9143
Variable 8 (Level 2)	-0.6768	1.7352
Variable 8 (Level 3)	-0.0439	1.6395
Variable 8 (Level 4)	0.1463	1.5358
Variable 9 (Level 1)	0.9761	2.3930
Variable 9 (Level 2)	6.5436	1.8193
Variable 9 (Level 3)	-1.5504	1.8527
Variable 9 (Level 4)	0.1047	2.0244
Variable 9 (Level 5)	-3.9386	1.7937
Subject: Variable 10 (Level 1)	Variable 11 (Level 1)	Variable 12 (Level 3)
Variable 3 (Level 1)	10.6802	3.2596
Variable 3 (Level 2)	-1.0290	3.7842
Variable 4 (Level 1)	-2.8612	2.2917
Variable 4 (Level 2)	3.9265	2.8934
Variable 4 (Level 3)	2.2427	2.3737
Subject: Variable 10 (Level 2)	Variable 11 (Level 1)	Variable 12 (Level 3)
Variable 3 (Level 1)	-6.2076	3.3642
Variable 3 (Level 2)	-8.7670	3.8463
Variable 4 (Level 1)	-2.9251	2.4657
Subject: Variable 10 (Level 2)	Variable 11 (Level 1)	Variable 12 (Level 3)
Variable 4 (Level 3)	-2.2077	2.3743
Subject: Variable 10 (Level 1)	Variable 11 (Level 2)	Variable 12 (Level 3)
Variable 3 (Level 1)	-3.3334	3.4665
Variable 3 (Level 2)	-0.3111	3.2650
Variable 4 (Level 1)	1.5131	2.4890
Variable 4 (Level 2)	-3.0345	3.0562
Variable 4 (Level 3)	0.2722	2.8300
Subject: Variable 10 (Level 2)	Variable 11 (Level 2)	Variable 12 (Level 3)
Variable 3 (Level 1)	6.5905	4.0386
Variable 3 (Level 2)	-5.3168	3.4549
Variable 4 (Level 1)	-3.5280	2.9663
Variable 4 (Level 2)	1.7056	2.9293
Variable 4 (Level 3)	2.2590	3.1780
Subject: Variable 10 (Level 1)	Variable 11 (Level 3)	Variable 12 (Level 3)
Variable 3 (Level 1)	8.1889	4.1429
Variable 3 (Level 2)	-1.5388	3.3333
Variable 4 (Level 1)	3.4338	2.6376
Subject: Variable 10 (Level 1)	Variable 11 (Level 3)	Variable 12 (Level 3)
Variable 4 (Level 3)	-1.1544	2.9885
Subject: Variable 10 (Level 2)	Variable 11 (Level 3)	Variable 12 (Level 3)
Variable 3 (Level 1)	-4.4243	4.0049
Variable 3 (Level 2)	-4.1349	3.1248

Variable 4 (Level 1)	1.0460	2.6550
Variable 4 (Level 2)	-4.4844	2.2843
Variable 4 (Level 3)	0.5046	2.6926
Subject: Variable 11 (Level 1) Variable 12 (Level 3)		
Variable 5 (Level 1)	5.3030	3.0278
Variable 5 (Level 2)	-8.1794	3.1335
Variable 6 (Level 1)	-0.8188	3.7810
Variable 6 (Level 2)	-2.5078	3.1514
Variable 6 (Level 3)	-2.6138	3.4600
Subject: Variable 11 (Level 2) Variable 12 (Level 3)		
Variable 5 (Level 1)	4.3331	3.1489
Variable 5 (Level 2)	-5.6142	3.1649
Variable 6 (Level 1)	-5.8804	3.1770
Variable 6 (Level 2)	5.4265	3.3006
Variable 6 (Level 3)	-2.1917	3.2156
Subject: Variable 11 (Level 3) Variable 12 (Level 3)		
Variable 5 (Level 1)	0.4305	2.9144
Variable 5 (Level 2)	-1.4620	3.0119
Variable 6 (Level 1)	14.3595	3.9254
Variable 6 (Level 2)	-5.2399	3.3099
Variable 6 (Level 3)	-11.2498	3.2212
Subject: Variable 12 (Level 3)		
Variable 7	-0.3839	0.6755
Variable 8 (Level 1)	2.7549	1.6017
Variable 8 (Level 2)	0.4377	1.8826
Variable 8 (Level 3)	-0.2261	1.9909
Variable 8 (Level 4)	-4.5051	1.5398
Variable 9 (Level 1)	-4.7091	2.1458
Variable 9 (Level 2)	3.7940	1.9872
Variable 9 (Level 3)	-1.7994	1.8614
Variable 9 (Level 4)	0.4480	1.9016
Variable 9 (Level 5)	-0.6047	2.4729
Fixed Effects		
Intercept	1.6433	2.4596
Variable 1 (Level 2)	-1.6224	0.8549
Variable 2 (Level 2)	-2.4817	1.1414
Variable 2 (Level 3)	0.4624	1.2133
Variance Components		
Estimate	Parameter	Subject
36.32491	Variable 3	Variables 10 11 12
12.45090	Variable 4	Variables 10 11 12
19.62767	Variable 5	Variables 11 12
40.53480	Variable 6	Variables 11 12
0.56320	Variable 7	Variables 12
5.81968	Variable 8	Variables 12
10.86069	Variable 9	Variables 12
SIGMA**2 = 0.00239		
-2LOG LIKELIHOOD = 608.19449		
