

NAG Library Function Document

nag_ranks_and_scores (g01dhc)

1 Purpose

nag_ranks_and_scores (g01dhc) computes the ranks, Normal scores, an approximation to the Normal scores or the exponential scores as requested by you.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
void nag_ranks_and_scores (Nag_Scores scores, Nag_Ties ties, Integer n,
                           const double x[], double r[], NagError *fail)
```

3 Description

nag_ranks_and_scores (g01dhc) computes one of the following scores for a sample of observations, x_1, x_2, \dots, x_n .

1. Rank Scores

The ranks are assigned to the data in ascending order, that is the i th observation has score $s_i = k$ if it is the k th smallest observation in the sample.

2. Normal Scores

The Normal scores are the expected values of the Normal order statistics from a sample of size n . If x_i is the k th smallest observation in the sample, then the score for that observation, s_i , is $E(Z_k)$ where Z_k is the k th order statistic in a sample of size n from a standard Normal distribution and E is the expectation operator.

3. Blom, Tukey and van der Waerden Scores

These scores are approximations to the Normal scores. The scores are obtained by evaluating the inverse cumulative Normal distribution function, $\Phi^{-1}(\cdot)$, at the values of the ranks scaled into the interval $(0, 1)$ using different scaling transformations.

The Blom scores use the scaling transformation $\frac{r_i - \frac{3}{8}}{n + \frac{1}{4}}$ for the rank r_i , for $i = 1, 2, \dots, n$. Thus the Blom score corresponding to the observation x_i is

$$s_i = \Phi^{-1}\left(\frac{r_i - \frac{3}{8}}{n + \frac{1}{4}}\right).$$

The Tukey scores use the scaling transformation $\frac{r_i - \frac{1}{3}}{n + \frac{1}{3}}$; the Tukey score corresponding to the observation x_i is

$$s_i = \Phi^{-1}\left(\frac{r_i - \frac{1}{3}}{n + \frac{1}{3}}\right).$$

The van der Waerden scores use the scaling transformation $\frac{r_i}{n + 1}$; the van der Waerden score corresponding to the observation x_i is

$$s_i = \Phi^{-1}\left(\frac{r_i}{n + 1}\right).$$

The van der Waerden scores may be used to carry out the van der Waerden test for testing for differences between several population distributions, see Conover (1980).

4. Savage Scores

The Savage scores are the expected values of the exponential order statistics from a sample of size n . They may be used in a test discussed by Savage (1956) and Lehmann (1975). If x_i is the k th smallest observation in the sample, then the score for that observation is

$$s_i = E(Y_k) = \frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{n-k+1},$$

where Y_k is the k th order statistic in a sample of size n from a standard exponential distribution and E is the expectation operator.

Ties may be handled in one of five ways. Let $x_{t(i)}$, for $i = 1, 2, \dots, m$, denote m tied observations, that is $x_{t(1)} = x_{t(2)} = \cdots = x_{t(m)}$ with $t(1) < t(2) < \cdots < t(m)$. If the rank of $x_{t(1)}$ is k , then if ties are ignored the rank of $x_{t(j)}$ will be $k+j-1$. Let the scores ignoring ties be $s_{t(1)}^*, s_{t(2)}^*, \dots, s_{t(m)}^*$. Then the scores, $s_{t(i)}$, for $i = 1, 2, \dots, m$, may be calculated as follows:

- if averages are used, then $s_{t(i)} = \sum_{j=1}^m s_{t(j)}^*/m$;
- if the lowest score is used, then $s_{t(i)} = s_{t(1)}^*$;
- if the highest score is used, then $s_{t(i)} = s_{t(m)}^*$;
- if ties are to be broken randomly, then $s_{t(i)} = s_{t(I)}^*$ where $I \in \{\text{random permutation of } 1, 2, \dots, m\}$;
- if ties are to be ignored, then $s_{t(i)} = s_{t(i)}^*$.

4 References

- Blom G (1958) *Statistical Estimates and Transformed Beta-variables* Wiley
 Conover W J (1980) *Practical Nonparametric Statistics* Wiley
 Lehmann E L (1975) *Nonparametrics: Statistical Methods Based on Ranks* Holden-Day
 Savage I R (1956) Contributions to the theory of rank order statistics – the two-sample case *Ann. Math. Statist.* **27** 590–615
 Tukey J W (1962) The future of data analysis *Ann. Math. Statist.* **33** 1–67

5 Arguments

- 1: **scores** – Nag_Scores *Input*
On entry: indicates which of the following scores are required.
scores = Nag_RankScores
 The ranks.
scores = Nag_NormalScores
 The Normal scores, that is the expected value of the Normal order statistics.
scores = Nag_BlomScores
 The Blom version of the Normal scores.
scores = Nag_TukeyScores
 The Tukey version of the Normal scores.
scores = Nag_WaerdenScores
 The van der Waerden version of the Normal scores.

scores = Nag_SavageScores

The Savage scores, that is the expected value of the exponential order statistics.

Constraint: **scores** = Nag_RankScores, Nag_NormalScores, Nag_BломScores, Nag_TukeyScores, Nag_WaerdenScores or Nag_SavageScores.

2: **ties** – Nag_Ties

Input

On entry: indicates which of the following methods is to be used to assign scores to tied observations.

ties = Nag_AverageTies

The average of the scores for tied observations is used.

ties = Nag_LowestTies

The lowest score in the group of ties is used.

ties = Nag_HighestTies

The highest score in the group of ties is used.

ties = Nag_RandomTies

The repeatable random number generator is used to randomly untie any group of tied observations.

ties = Nag_IgnoreTies

Any ties are ignored, that is the scores are assigned to tied observations in the order that they appear in the data.

Constraint: **ties** = Nag_AverageTies, Nag_LowestTies, Nag_HighestTies, Nag_RandomTies or Nag_IgnoreTies.

3: **n** – Integer

Input

On entry: n , the number of observations.

Constraint: $n \geq 1$.

4: **x[n]** – const double

Input

On entry: the sample of observations, x_i , for $i = 1, 2, \dots, n$.

5: **r[n]** – double

Output

On exit: contains the scores, s_i , for $i = 1, 2, \dots, n$, as specified by **scores**.

6: **fail** – NagError *

Input/Output

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT_ARG_LT

On entry, **n** = $\langle value \rangle$.
 Constraint: **n** ≥ 1 .

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

For **scores** = Nag_RankScores, the results should be accurate to ***machine precision***.

For **scores** = Nag_SavageScores, the results should be accurate to a small multiple of ***machine precision***.

For **scores** = Nag_NormalScores, the results should have a relative accuracy of at least $\max(100 \times \epsilon, 10^{-8})$ where ϵ is the ***machine precision***.

For **scores** = Nag_BломScores, Nag_TukeyScores or Nag_WaerdenScores, the results should have a relative accuracy of at least $\max(10 \times \epsilon, 10^{-12})$.

8 Parallelism and Performance

`nag_ranks_and_scores` (g01dhc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

If more accurate Normal scores are required `nag_normal_scores_exact` (g01dac) should be used with appropriate settings for the input argument **etol**.

10 Example

This example computes and prints the Savage scores for a sample of five observations. The average of the scores of any tied observations is used.

10.1 Program Text

```
/* nag_ranks_and_scores (g01dhc) Example Program.
*
* NAGPRODCODE Version.
*
* Copyright 2016 Numerical Algorithms Group.
*
* Mark 26, 2016.
*/
#include <nag.h>
```

```

#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0, i, n;
    NagError fail;
    double *r = 0, *x = 0;

    INIT_FAIL(fail);

    printf("nag_ranks_and_scores (g01dhc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT " ", &n);
#else
    scanf("%" NAG_IFMT " ", &n);
#endif
    if (n >= 1) {
        if (!(r = NAG_ALLOC(n, double)) || !(x = NAG_ALLOC(n, double)))
        {
            printf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
    }
    else {
        printf("Invalid n.\n");
        exit_status = 1;
        return exit_status;
    }
    for (i = 1; i <= n; ++i)
#ifdef _WIN32
    scanf_s("%lf ", &x[i - 1]);
#else
    scanf("%lf ", &x[i - 1]);
#endif

/* nag_ranks_and_scores (g01dhc).
 * Ranks, Normal scores, approximate Normal scores or
 * exponential (Savage) scores
 */
    nag_ranks_and_scores(Nag_SavageScores, Nag_AverageTies, n, x, r, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ranks_and_scores (g01dhc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    printf("The Savage Scores : \n");
    printf(" (Average scores are used for tied observations)\n\n");
    for (i = 1; i <= n; ++i)
        printf("%10.4f\n", r[i - 1]);
END:
    NAG_FREE(r);
    NAG_FREE(x);
    return exit_status;
}

```

10.2 Program Data

```
nag_ranks_and_scores (g01dhc) Example Program Data
5
2 0 2 2 0
```

10.3 Program Results

```
nag_ranks_and_scores (g01dhc) Example Program Results
```

The Savage Scores :
(Average scores are used for tied observations)

```
1.4500
0.3250
1.4500
1.4500
0.3250
```
